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VISCOUS DISSIPATION IN JETS

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On the basis of the boundary-layer equations we consider self-similar flow regimes of a jet with an exponential dependence of the viscosity on temperature.

We consider an incompressible fluid jet ejected from a circular nozzle with an initial temperature not equal to the temperature of the surrounding medium. It is assumed that heat is produced in the flow because of viscous dissipation, and that the temperature dependence of the viscosity is given by

$$\mu = \mu_0 \exp(E/RT). \tag{1}$$

This relation is valid for the viscosities of condensed media in the temperature region of practical interest.

It was observed in [1] that for a given pressure drop there was a sharp change in the thermal behavior of the flow of a Newtonian fluid in an infinite pipe for $\chi > \chi_{cr}$, if the temperature dependence of the viscosity is given by (1). This was called a hydrodynamic thermal explosion.

In the present paper we study the possibility of a similar effect in an incompressible fluid jet.

The equations of motion and the heat equation in the boundary-layer approximation for the flow of the jet have the form [2]:

$$\begin{gathered} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(\nu y\frac{\partial u}{\partial y}\right), \\ u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{a}{y}\frac{\partial}{\partial y}\left(y\frac{\partial T}{\partial y}\right) + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \\ \frac{\partial}{\partial x}(yu) + \frac{\partial}{\partial y}(yv) = 0. \end{gathered} \tag{2}$$

With the boundary conditions:

$$\frac{\partial u}{\partial y} = 0, \frac{\partial T}{\partial y} = 0, \ v = 0, \ y = 0; \ T \to T_\infty, \ u \to 0, \ y \to \infty. \tag{3}$$

In the self-similar formulation of the problem the initial conditions are replaced by the integral relations

$$I = I_0 = \pi\rho u_0^2 d^2/4, \ Q = Q_0 = \pi\rho c_p u_0 T_0 d^2/4. \tag{4}$$

The subscript zero indicates that the corresponding quantity is evaluated at the nozzle cross section.

We apply the method of local similarity [3]:

$$u = u_m(x) f'(\varphi)/\varphi, \ \Delta T = \Delta T_m(x)\,\theta(\varphi), \ \varphi = y/\delta(x). \tag{5}$$

Using the Frank–Kamenetskii expansion [4], we transform the system of equations (2) to the following form, using the method of local similarity:

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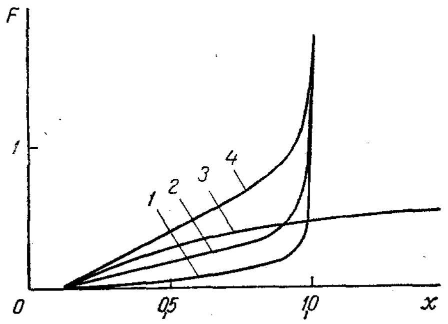

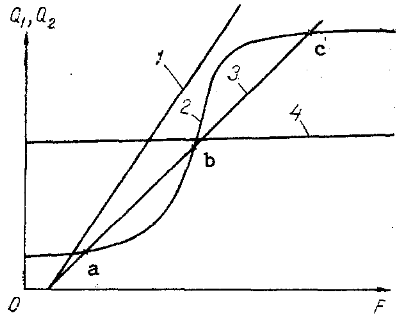

Fig. 1 Fig. 2

Fig. 1. Variation of the heat flux F with distance along the axis of the jet: 1) mb = 9, b = 0.05; 2) mb = 9, b = 0.1; 3) mb = 10, b = 0.2; 4) mb = 9, b = 0.2.

Fig. 2. Dependence of the heat removed Q_1 and heat production Q_2 on the heat flux F.

$$u_m^2\delta^2 = 1/(2\gamma_1),$$
$$u_m\delta^2 = \mathrm{Sv}\exp\left(-\frac{\Delta T_m}{1+\beta\Delta T_m}\right)x, \qquad (6)$$
$$\frac{dF}{dx} = a_4\,\mathrm{Br}\exp\left(-\frac{\Delta T_m}{1+\beta\Delta T_m}\right)\frac{u_m^2}{\lambda_1},$$
$$a_4 = 0.21;\ \gamma_1 = 0.09;\ \lambda_1 = 0.15;\ \lambda = 12.$$

We used the profile $1 - 3\phi^2 + 2\phi^3$ [5] for the stream function and the П-profile for the temperature in performing the integration at the cross section of the nozzle. The П-profile for the temperature in a jet is valid when $\mathrm{Pr} \gg 1$, which is satisfied in condensed media over a wide range of the parameters of the problem. We have the boundary condition $F(x_0) = 1$; the mixture is heated with respect to the surrounding medium. Here x_0 is the polar distance of the jet. In [3] it was found that $x_0 \simeq -0.15$ in a numerical solution for the case of a non-self-similar jet.

The solution of the hydrodynamic part of the above system of equations has the form

$$\delta = (2\gamma_1)^{1/2}\mathrm{Sv}\exp\left(-\frac{\Delta T_m}{1+\beta\Delta T_m}\right)x, \qquad (7)$$
$$u_m = \frac{\exp\left(\dfrac{\Delta T_m}{1+\beta\Delta T_m}\right)}{2\gamma_1\mathrm{Sv}\,x}.$$

Expressing the temperature ΔT_m in terms of the heat flux F, the axial component of the velocity u_m, and the thickness of the jet δ, we have the following heat transport equation

$$\frac{dF}{dx} = \frac{a_4\mathrm{Br}\exp\left[\dfrac{F}{\mathrm{Sv}(x+\beta F)}\right]}{\lambda_1(2\gamma_1)^2\mathrm{Sv}^2x^2}. \qquad (8)$$

In evaluating the exponential on the right hand side of this equation, we used the following asymptotic expansions for the velocity and thickness of the jet

$$u_m \sim (2\gamma_1\mathrm{Sv}\,x)^{-1} + O(\Delta T_m),$$
$$\delta \sim (2\gamma_1)^{1/2}\mathrm{Sv}\,x + O(\Delta T_m). \qquad (9)$$

To study the possibility of a hydrodynamic heat explosion in a jet we consider the asymptotic properties of a heat explosion, which assume a finite combustion path x_c, the analog of an induction period, and also a strong dependence of the viscosity of the fluid on temperature, $\beta \ll 1$:

$$\int_{F(x_0)}^{\infty} \exp\left(-\frac{F}{\mathrm{Sv}\,x}\right) d\left(\frac{F}{\mathrm{Sv}}\right) = \int_{x_0}^{x_B} \frac{a_4 \mathrm{Br}\, dx}{\lambda_1 (2\gamma_1)^2 \mathrm{Sv}^3 x^2}. \tag{10}$$

The quantity x on the left hand side of this equation is treated as a parameter. This assumption can be used because of the asymptotic properties of heat explosions; the growth of F in the immediate neighborhood of x_c is determined by the following region of the parameters: $\mathrm{Br} \gg \mathrm{Sv}^2$; $\mathrm{Sv}\, x_c \ll 1$. The choice of this region and the possibility of a sharp change in the thermal behavior described by equations analogous to (8) are shown below for an idealized example. The possibility of treating the variable x as a parameter was stimulated by the ideas of [6].

Integration of (10) gives the result

$$Z_1 x_c^2 + Z_2 x_c + Z_3 = 0,$$
$$Z_1 = -x_0 \exp[-F(x_0)/(\mathrm{Sv}\, x_0)], \quad Z_2 = -\frac{a_4 \mathrm{Br}}{\lambda_1 (2\gamma_1)^2 \mathrm{Sv}^3}, \quad Z_3 = -Z_2 x_0. \tag{11}$$

We can now determine the intervals of the viscous dissipation parameter Br corresponding to the regions of existence of the solution:

$$\begin{gathered} \text{for } \mathrm{Br} > \mathrm{Br}_{cr} \text{ (two solutions)}, \\ \text{for } \mathrm{Br} = \mathrm{Br}_{cr} \text{ (one solution)}, \\ \text{for } \mathrm{Br} < \mathrm{Br}_{cr} \text{ (no solution)}, \\ \mathrm{Br}_{cr} = [4\lambda_1 (2\gamma_1)^2 \mathrm{Sv}^3 x_0^2]/\{a_4 \exp[F(x_0)/(\mathrm{Sv}\, x_0)]\}, \end{gathered} \tag{12}$$

and therefore a hydrodynamic heat explosion does not occur.

The analytical approach to the problem yields an expression for the combustion path, which is an important characteristic of the motion of a fluid with a sharp change in its thermal behavior:

$$(x_c)_{1,2} = \frac{-Z_2 \pm \sqrt{Z_2^2 - 4Z_1 Z_2}}{2Z_1}. \tag{13}$$

We consider the following idealized example in order to show the possibility of a hydrodynamic heat explosion following from (8):

$$\frac{dF}{dx} = m\frac{\exp(F/u_m\delta^2)}{x^2}, \quad F(x_0) = F_0. \tag{14}$$

We put $u_m\delta^2$ = const = b. This relation is obtained by integrating the equation of continuity using the boundary conditions of an axisymmetric jet. Then (14) reduces to the form

$$\frac{dF}{dx} = m\frac{\exp(F/b)}{x^2}. \tag{15}$$

Integration of this equation gives

$$F = b \ln\left[\frac{mb}{1/x - 1/x_0 + mb\exp(-F_0/b)}\right]. \tag{16}$$

In order to illustrate (16) we put $F(x_0) = 0$, $x_0 = 0.1$; this corresponds to a cold mixture where the initial temperature of the jet is equal to that of the surrounding medium. Curves are shown in Fig. 1 for several values of the parameters m and b. Comparison of curves 1 and 4 shows the region of applicability of treating the variable x as a parameter in the integration of (10): $b \ll 1$, $m \gg 1$. This corresponds to an insignificant increase in F over the large region from zero to x_c and then a sharp change in F near x_c.

We analyze the obtained results, emphasizing the features of a hydrodynamic heat explosion in a jet in comparison with the analogous phenomenon in a pipe. We consider the asymptotic forms of the low and high temperature regimes: $\beta \gg 1$, $\beta \ll 1$, respectively. The argument of the exponentials in (6) has the form

$$\beta \gg 1: \quad \frac{\Delta T_m}{1 + \beta\Delta T_m} \sim \frac{1}{\beta} + O\left(\frac{1}{\beta^2}\right),$$

$$\beta \ll 1: \quad \frac{\Delta T_m}{1+\beta\Delta T_m} \sim \Delta T_m + O(\beta). \tag{17}$$

Using (17) we obtain expressions for the velocity and thickness of the jet in the low and high temperature regimes:

$$\beta \gg 1: \ \delta \sim (2\gamma_1)^{1/2}\mathrm{Sv}\, e^{-\frac{1}{\beta}} x + O(\beta^{-2}), \ u_m = \frac{e^{\frac{1}{\beta}}}{2\gamma_1 \mathrm{Sv}\, x} + O(\beta^{-2}),$$
$$\beta \ll 1: \ \delta \sim (2\gamma_1)^{1/2}\mathrm{Sv}\, e^{-\Delta T_m} x + O(\beta), \ u_m \sim \frac{e^{\Delta T_m}}{2\gamma_1 \mathrm{Sv}\, x} + O(\beta). \tag{18}$$

These relations can be used to determine the nature of the hydrodynamics of the jet in the presence of viscous dissipation when the viscosity of the fluid has a strong dependence on temperature.

In the low-temperature regime viscous dissipation does not significantly change the nature of the variation of velocity and thickness along the axis of the jet: $u_m \sim 1/x$ (Br = 0), $\delta \sim x$ (Br = 0). Heating of the fluid leads to an increase in the flow velocity and a decrease in the thickness of the jet; ejection of the surrounding medium decreases for a given impulse of the jet.

We consider now a numerical estimate of the condition for a hydrodynamic heat explosion for the flow of glycerin, whose viscosity depends strongly on temperature. According to [7], the viscosity of glycerin as a function of temperature is well approximated by the expression $\mu = 2.69 \cdot 10^{-7}\, e^{3337/T}$ Pa-sec. Using the following values of the parameters: $\rho = 1.26 \cdot 10^3$ kg/m³; $T_\infty = 293$°K; $u_0 = 0.3$ m/sec; $d = 10^{-2}$ m, we obtain Br = 0.77 for the viscous disdipation parameter, which determines the condition for a hydrodynamic heat explosion in glycerin.

We return to a qualitative interpretation of the production of a hydrodynamic heat explosion in a fluid jet by a mechanism which is well known in the theory of heat explosions. Figure 2 shows curves of the heat removed by convection and the heat production due to viscous dissipation. Curve 1 corresponds to a larger quantity of heat removed by convection than curve 3; the heat production in the low-temperature regime is characterized by curve 4 and in the high-temperature regime by curve 2. In the low-temperature flow regime there is always one point of intersection of the two heat curves. A hydrodynamic heat explosion does not occur in this case and the flow is steady. The high-temperature flow regime is characterized by several points of intersection of the two heat curves (from one to three points, depending on the quantity heat removed). Three intersection points are possible for a low value of the heat removed and the middle point represents an unstable state (curve 3). The point a corresponds to the low-temperature regime and the point c to the high-temperature regime; the transition from the low to the high-temperature state corresponds to a change in the velocity from a large value to a small value.

At small flow velocities heat cannot flow away and a steady flow regime becomes impossible, i.e., a hydrodynamic heat explosion arises.

Finally we point out the difference between a hydrodynamic heat explosion in a jet and the analogous phenomenon in a pipe. As noted in [1], the essential difference between a hydrodynamic heat explosion for a fluid flowing in a pipe and a heat explosion of chemical origin is that in the former case there is no physical analog of the heat of the reaction. But there is such an analog for a hydrodynamic heat explosion in a fluid jet: the degree of ejection of the surrounding medium, characterized by the entrainment parameter of the fluid Sv. The cause of the ejection is the viscosity which has two effects in a jet. First it causes shear stresses, like the viscosity in a hydrodynamic heat explosion in a pipe. Second it draws the surrounding medium into the jet. The smaller this effect, the larger the "heat" effect of the hydrodynamic heat explosion in an incompressible fluid jet.

NOTATION

μ, dynamical viscosity; E, activation energy; R, gas constant; T, temperature of the fluid in the jet; v, longitidinal and transverse components of the velocity; x, y, longitudinal and transverse coordinates; ν, kinematic viscosity; $\underline{a}$, thermal diffusivity; c_p, heat capacity at constant pressure; T_∞, temperature of the surrounding medium; I, impulse of the jet; Q, heat flux; ρ, density; d, nozzle diameter; ϕ, similarity variable; δ, thickness of the jet; f, stream function; θ, dimensionless temperature, $\theta = E(T - T_\infty)/TR_\infty^2$; Br, Brickman number, $Br = \mu u_0^2 d \exp(E/RT_\infty)/Q^*$, $Q^* = \pi\rho c_p u_0 d^2/4$; Sv, entrainment parameter of the fluid, $Sv = \mu \exp(E/RT_\infty)/\rho u_0 d$, $\beta = RT_\infty/E$; F, dimensionless heat flux, $F^* = \pi\rho c_p u_m \Delta T_m \delta^2$; Pr, Prandtl number, $Pr = \nu/a$; $Q_1 \sim (F - F_0)/d$; $Q_2 \sim \exp(F/(x_0 + \beta F)Sv)/x_B^2$.

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DETERMINING RHEOLOGICAL PARAMETERS FOR A DISPERSION SYSTEM BY ROTATIONAL VISCOMETRY

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An algorithm has been devised for inverse treatment in rotational viscometry subject to a priori uncertainty over the model. A model class has been formulated for rheologically stationary systems.

Dispersion systems are widely used, and determining their rheological characteristics is important in data support to optimum management.

Some methods of processing data from rotational rheometry [1-3] make inadequate use of the information from experiment to derive models and evaluate parameters. Also, simplified methods are usually used to evaluate rheological characteristics for nonlinear viscoplastic media [1, 3], and these may give substantial errors in inverse treatments.

One can process such data from an equation describing Couette flow in a gap between coaxial cylinders [1]:

$$\omega = \frac{1}{2}\int_a^\tau \frac{\dot\gamma(\xi)}{\xi}\,d\xi. \tag{1}$$

The relation between the stresses on the outer and inner cylinders is

$$a = \begin{cases} \alpha^2\tau, & \text{if } \tau \geqslant \tau_0/\alpha^2; \\ \tau_0, & \text{if } \tau \in]\tau_0, \tau_0/\alpha^2[, \end{cases}$$

where τ_0 is the dynamic shear stress (yield point), $\alpha = R_1/R_2$, and R_1 and R_2 are the radii of the inner and outer cylinders.

An inverse rheometric treatment involves choosing the state index $\hat\nu$ for the medium from a certain class ϑ of rheologically stationary models known a priori and then estimating the parameter vector $\hat p_\nu$ for that model. The ϑ class can be formed from the following models: Newtonian ($\nu = 1$) – $\dot\gamma = \tau/\mu$, Shvedov–Bingham ($\nu = 2$) – $\dot\gamma = (\tau-\tau_0)/\mu$, Ostwald ($\nu = 3$) – $\dot\gamma = (\tau/k)^{1/n}$, Herschel–Bulkley ($\nu = 4$) – $\dot\gamma = ((\tau-\tau_0)/k)^{1/n}$, Schulman–Casson ($\nu = 5$) – $\dot\gamma = (\tau^{1/n} - \tau_0^{1/n})^n/\mu$, etc. Here μ, τ_0, k, n are the rheological parameters.

Statistical methods are applied to treating the data [4], on the assumption that the discrepancy between the measurement vector τ having components $\{\tau_i\}$, $i \in \overline{1, N}$ and the theoreti-

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